AEES Distance Learning Programme

Class IX Mathematics

Chapter 12- Heron’s Formula

Module 1 / 2

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In this module we will study the following points

1. Introduction
2. Area of triangle by heron’s formula

3 Practice Examples

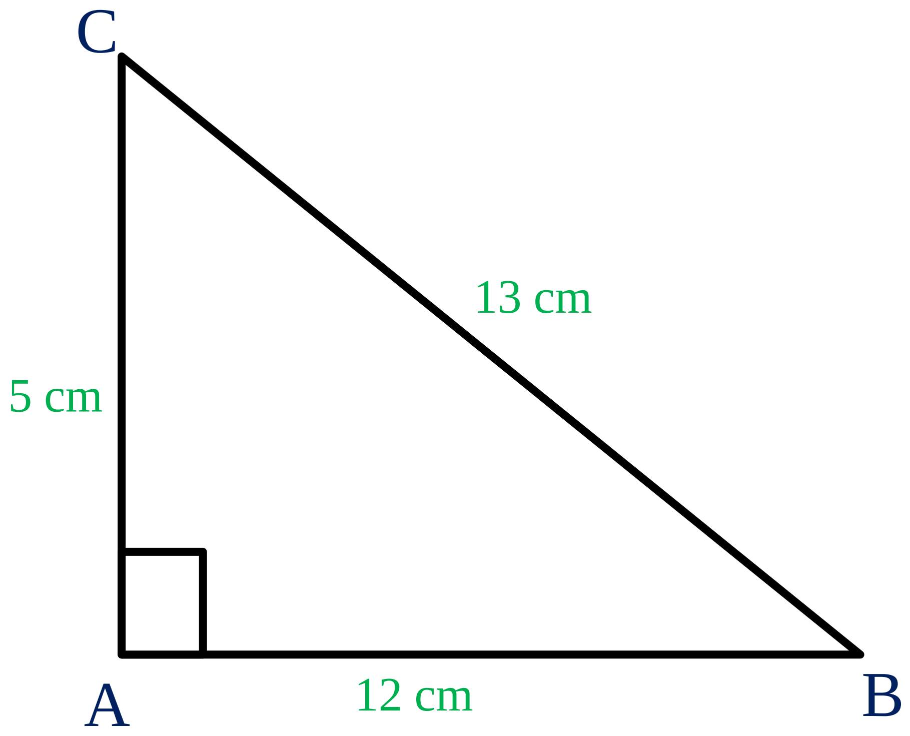
Introduction

You have studied in earlier classes about the figures like triangles, squares, rectangles and quadrilaterals , you have also learnt to find the areas and perimeters of these figures

You know that

Area of a triangle = $\frac{1}{2}\ X\ Base\ X\ Height$

For example , The sides of the right angled Δ ABC are 5 cm, 12 cm and 13 cm



base = 12 cm

height = 5 cm

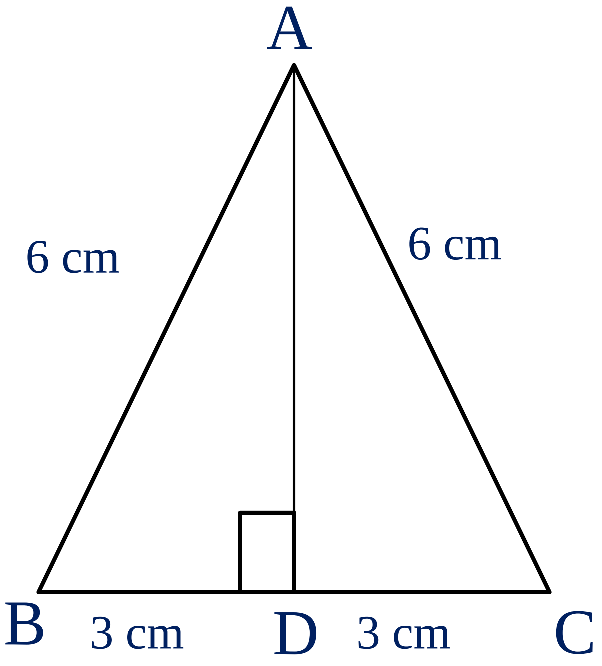
$\therefore$ Ar(Δ ABC) = ½ x b x h

= ½ x 12 x 5 cm^2

= 30 cm^2

We could also take 5 cm as the base and 12 cm as the height.

Area of an equilateral triangle ABC



We Need its height , if you join the midpoint of BC and vertex A we get a right angled Δ ADB and Δ ADC, and height AD

By using Pythagoras theorem. We can find the length of AD.

In Δ ADB

$$AB^2 = BD^2 + AD^2$$

$$\therefore\ AD^2 = AB^2 - BD^2$$

$$= 6^2 - 3^2$$

$$= 36 - 9$$

$$AD^2 = 27$$

$$AD = \sqrt{27} = 3\sqrt{3} \text{ cm}$$

Then

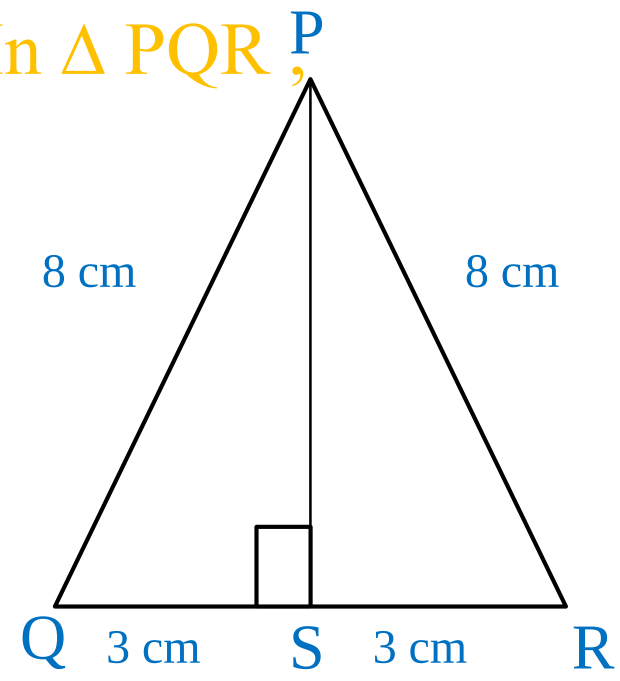
$$ar(\Delta ABC) = \tfrac{1}{2} \times b \times h$$

$$= \tfrac{1}{2} \times 6 \times 3\sqrt{3}$$

Area of an isosceles triangle

We can calculate the area of an isosceles triangle PQR with the help of above formula, here also we need to find the height of the triangle.

eq. In Δ PQR ,



PQ=PR=8cm
and QR=6cm

Draw the perpendicular PS from P to QR,PS divides the base QR into two equal parts this is possible for equilateral triangle and isosceles triangle

In Δ PQS , by Pythagoras theorem,

$PQ^2 = QS^2 + PS^2$

$8^2 = 3^2 + PS^2$

$PS^2 = 8^2 - 3^2$

$= 64 - 9$

$PS^2 = 55$

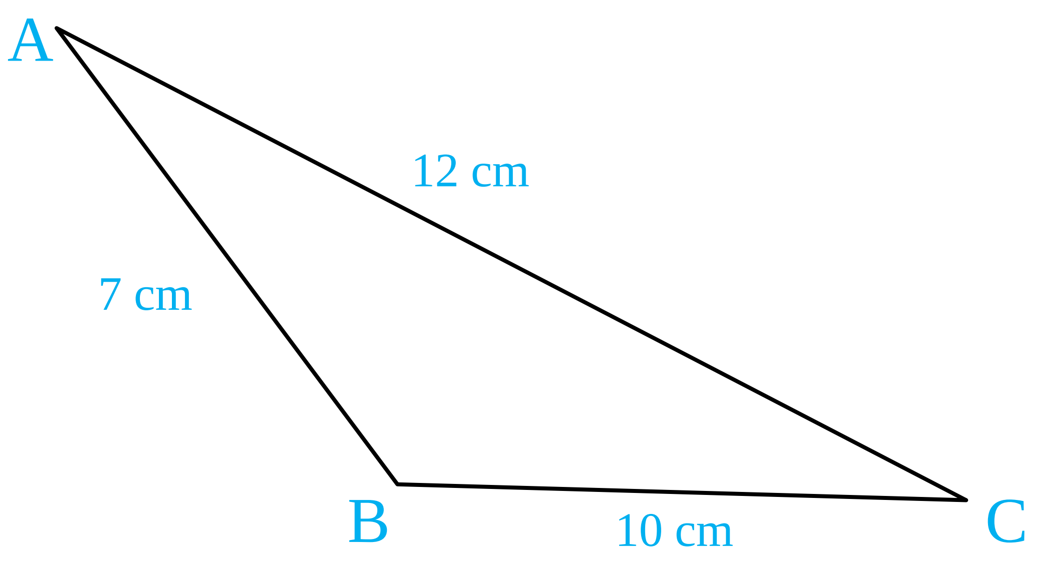
$PS = \sqrt{55}$ (height of Δ PQR)

$\therefore$ ar Δ ABC $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times \sqrt{55}\ cm^2$

$= 3\sqrt{55}\ cm^2$

To find the area of a Scalene triangle



You will have to calculate its height
We do not have any clue to find the height
Then, how to find the area of a triangle in terms of the lengths of its three sides?

Area of a triangle by Heron's Formula

Heron was born in about 10AD possibly in Alexandria in Egypt. He worked in applied mathematics. His works on mathematical and physical subjects are so numerous and varied that he is considered to be an encyclopedic writer in these fields. His geometrical works deal largely with problems on mensuration written in three books. Book I deals with the area of squares, rectangles, triangles, trapezoids (trapezia), various other specialised quadrilaterals, the regular polygons, circles, surfaces of cylinders, cones, spheres etc. In this book, Heron has derived the famous formula for the area of a triangle in terms of its three sides.

Heron gave the famous formula to find the area of a triangle in terms of its three sides is also known as Heron's Formula

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = $\frac{a+b+c}{2}$ is the semi perimeter and a , b & c are sides of the triangle

This formula is helpful where it is not possible to find the height of the triangle easily

Let us apply this formula to calculate the area of the triangle with three sides

Example 1

Find the area of a triangle whose sides are 13 cm , 14 cm & 15 cm

Soln.

a = 13 cm
b = 14 cm
c = 15 cm

$S = \frac{a+b+c}{2} = \frac{13+14+15}{2} = 21\text{cm}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{21(21-13)(21-14)(21-15)}$

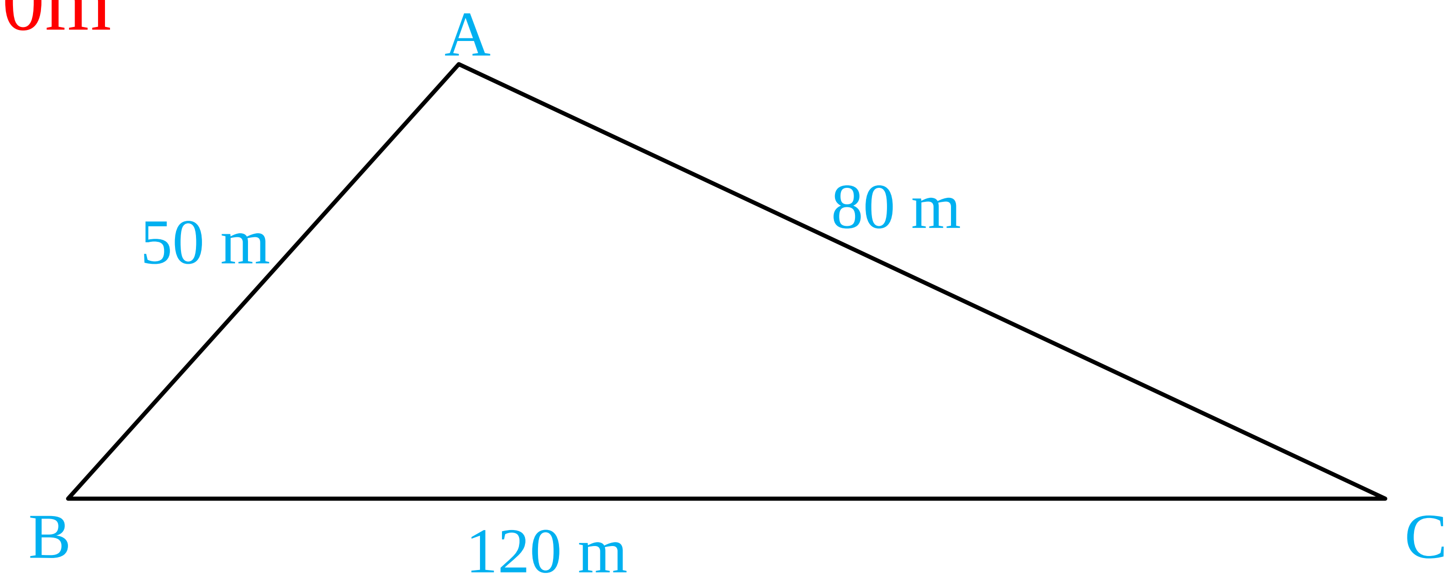
$A = \sqrt{21 * 8 * 7 * 6}$

$A = \sqrt{7 * 3 * 8 * 7 * 2 * 3}$

$A = \sqrt{7^2 * 4^2 * 3^2}$

$A = 7 * 4 * 3 = 84\ cm^2$

Ex 2. Find the area of a triangular park ABC with sides 120 m,80 m 50m



Solution:

S = $\frac{120+80+50}{2}$ = 125m

A = $\sqrt{s(s-a)(s-b)(s-c)}$

A = $\sqrt{125(125-120)(125-80)(125-50)}$

A = $\sqrt{125*5*45*75}$

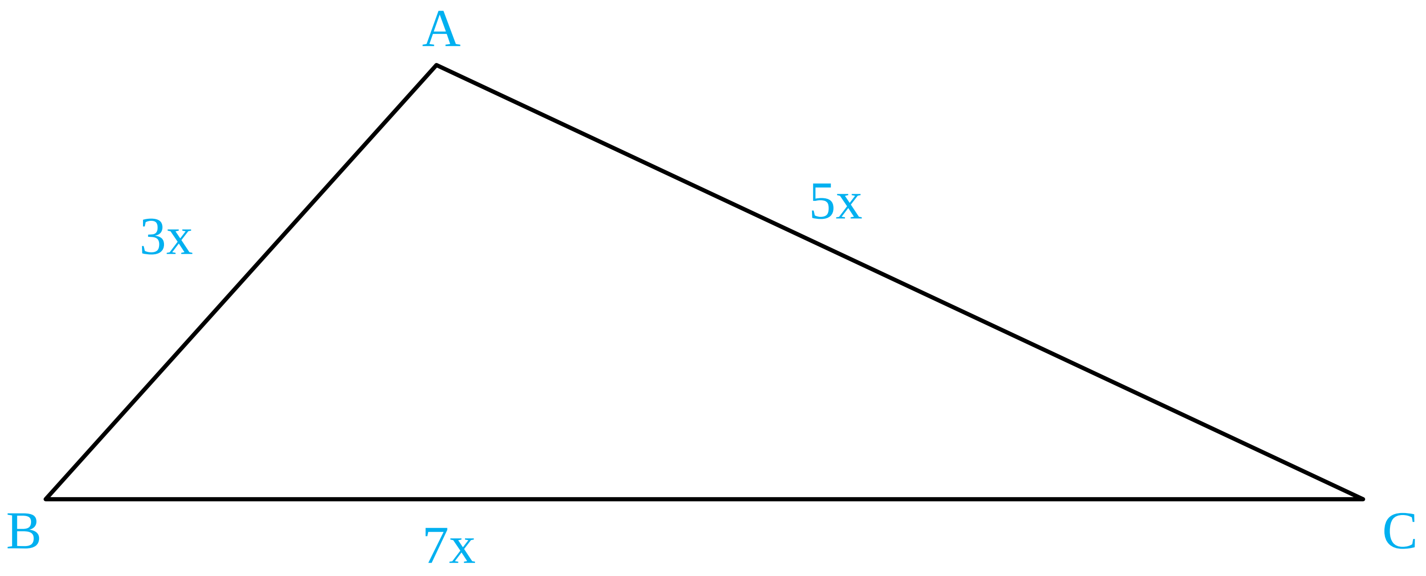
A = $375\sqrt{15}$ m^2

Therefore ,the area of the park = $375\sqrt{15}$ m^2

Ex 3. The sides of a triangular plot are in the ratio of 3:5:7 and its perimeter is 300 m, Find its area.

Solution:

Lets the sides be 3x ,5x and 7x



$S = \frac{P}{2}$

P= 2xS

$300 = \cancel{2} \times \frac{3x + 5x + 7x}{\cancel{2}}$

$\therefore 15x = 300$

$x = 20$

So, the sides are 3 x 20m = 60m = a

5 x 20m = 100m = b

7 x 20m = 140m = c

.: S = $\frac{60+100+140}{2}$ = 150 m

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{150(150-60)(150-100)(150-140)}$

$A = \sqrt{150 X 90 \; X \; 50 \; X \; 10}$

Area = $1500\sqrt{3}\ m^2$

Ex 4. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Solution.

Let the length of the unequal side be x

:. P= 12+12+x

x + 24 = 30

x = 6

$$S = \frac{12+12+6}{2} = 15 \text{ cm}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{15(15-6)(15-12)(15-12)}$$

$$A = \sqrt{15 * 9 * 3 * 3}$$

$$A = 9\sqrt{15}\ \text{cm}^2$$

THANK YOU